

Diagnosing Craquelure: A Mechanistic Case Study

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Abstract:

Background: Paintings, as complex multi-layered composite structures, exhibit fracture patterns—collectively termed craquelure—that parallel the clinical presentation of stress-induced material failure. Understanding the mechanical etiology of these cracks is essential for implementing effective preventive conservation strategies.

Case Presentation: This study presents a comprehensive mechanistic case analysis of a representative two-layer painting system comprising a glue-sized canvas mounted on a flexible wooden stretcher with a brittle chalk-glue ground layer, simulating a historical pictorial surface. Using three-dimensional finite element modeling, we investigated stress development and crack initiation under desiccation induced by substantial drops in relative humidity (RH).

Results: The model reveals a critical pathological mechanism: when a realistic flexible stretcher is incorporated, hygroscopic canvas shrinkage induces inward bar deflection, concentrating high tensile stresses at the painting's corners—explaining the preferential formation of corner cracks oriented perpendicular to the diagonal. The study quantifies the critical crack spacing-to-ground thickness ratio (S/t_g) at fracture saturation for various RH reductions. A diagnostic double Lorentz function was developed to characterize stress redistribution around individual cracks, enabling accurate prediction of crack positions without computationally expensive full-field simulations. Model predictions for crack spacing (5–6 mm) and damaged zone extent aligned closely with observations from a laboratory mock-up painting.

Discussion: The findings delineate two distinct pathological pathways: humidity-induced corner cracking versus long-term cumulative drying shrinkage of oil-based paint layers in central zones. This differential diagnosis has direct clinical implications for conservation, suggesting that targeted microclimate control at the painting's periphery may provide a highly efficient preventive strategy.

Conclusion: This work establishes a robust, evidence-based diagnostic framework for predicting humidity-induced damage in canvas paintings, offering practical tools for risk assessment and informed preventive conservation.

Keywords: Canvas paintings; Craquelure; Finite element modeling; Humidity-induced fracture; Preventive conservation; Mechanistic case study

1.Introduction

The catastrophic failure of load-bearing biomedical devices—ranging from fractured ceramic femoral heads in total hip arthroplasties and cracked polymethyl methacrylate (PMMA) bone cement mantles to the stress-corrosion cracking of metallic cardiovascular stents and the environmental stress cracking (ESC) of polymer catheter hubs—remains a persistent and high-stakes challenge in modern clinical practice. While such failures are conventionally attributed to isolated fatigue, manufacturing defects, or aggressive biological fluids, they fundamentally represent a universal mechanical pathology: the fracture of

a brittle or constrained layer adhered to a dimensionally active or cyclically loaded substrate, governed by shear-lag stress transfer, critical defect spacing, and fracture saturation thresholds. Paradoxically, an ideal long-term, century-scale observational platform for studying this exact mechanical phenomenon is not found in a biomechanics laboratory, but within the layered composite structures of historical canvas paintings. The evolution of craquelure—a network of fine cracks driven by hygroscopic shrinkage of a flexible canvas support against a brittle chalk-glue ground layer—recapitulates, with striking fidelity, the identical fracture

mechanics observed in degrading implant coatings and failing surgical materials. By treating craquelure as a "clinical case" of progressive material failure, this study develops validated three-dimensional finite element models and closed-form analytical solutions—including critical crack spacing-to-thickness ratios (S/tg), shear-lag equilibrium equations, and double Lorentz stress redistribution functions—to diagnostically predict how sequential cracking relieves stress, how layer thickness governs fracture susceptibility, and when fracture saturation stabilizes the system. These heritage-science derived predictive tools offer a unique and powerful translational opportunity: they provide a physically-based, experimentally-validated framework for quantitatively assessing the risk of interfacial delamination and coating fracture in orthopedic implants, optimizing layer thickness ratios in load-bearing prosthetics to suppress stress concentrations, and designing more fatigue-resistant surgical materials by understanding how controlled stress redistribution through sequential micro-cracking can prevent catastrophic, single-event failure. In essence, the diagnostic framework developed for aging canvas paintings may inform the next generation of durable, fracture-resistant biomedical devices. The study of craquelure—the network of fine cracks that develop on painted surfaces—represents a crucial intersection between art conservation, materials science, and mechanical engineering. These crack patterns, once viewed primarily as signs of deterioration, have emerged as valuable sources of information about artistic materials, environmental history, and the mechanical behavior of composite

structures. This literature review synthesizes research from the past five decades, tracing the evolution from descriptive documentation to sophisticated computational modeling of crack formation and propagation in paintings [1-10].

Crack systems in paintings originate from a combination of material ageing, drying shrinkage, environmental variability, physical handling, and structural instabilities. These fracture networks—collectively termed craquelure—reflect the particular materials and methods employed by an artist, and their characteristics are therefore valuable indicators of a work's authenticity and provenance. At the same time, crack patterns alter the visual appearance of a painting and, when actively propagating, signal ongoing material deterioration. Preventing new fracture formation and limiting the widening of existing cracks are therefore central goals in collection management and preventive conservation [11-20].

Figure 1 is an example of a craquelure pattern on a painted surface, illustrating the network of fine cracks that develop from material ageing and environmental stress. This image provides a visual reference for the physical phenomenon being investigated, showing the complex, interconnected fracture network that can form over time. The morphology of these cracks—their orientation, spacing, and intersection angles—serves as a key source of information about the artwork's material composition and environmental history, and it is the ultimate target of the predictive models developed in this study.



Figure 1: Sample Crack Pattern.

The systematic study of craquelure began with foundational work by Stout [1], who developed one of the first trial indices for classifying laminal disruption in paintings. This pioneering effort established a vocabulary and framework for describing crack patterns, recognizing that different types of craquelure might correspond to different causes or material conditions. Stout's work provided conservators with a tool for documenting and comparing crack phenomena across different artworks and contexts.

Building upon this foundation, Bucklow [2] advanced the field by developing a more refined system for describing craquelure patterns. His work emphasized that crack networks possess distinctive morphological

characteristics that could be systematically categorized. Bucklow's classification system considered factors such as crack orientation, spacing, intersection angles, and network topology, establishing that these features might serve as "fingerprints" revealing information about an artwork's materials, age, and environmental history. This descriptive approach laid the groundwork for subsequent analytical and computational investigations.

Early systematic efforts to classify craquelure were made by Stout [1] and Bucklow [2], who demonstrated that distinct crack morphologies are associated with specific artistic traditions and historical periods. Two principal cracking mechanisms were identified: drying cracks, arising

from the volumetric contraction of materials during or after application; and ageing cracks, driven by cyclic dimensional changes in the substrate induced by fluctuating ambient humidity [3,4]. In drying-type cracking, progressive loss of low-molecular-weight components from oil-based binders leads to the development of internal voids and cumulative shrinkage strains that can persist for decades or even centuries [5,6]. In humidity-induced cracking, glue sizing or wax-resin consolidants respond to moisture fluctuations far more strongly than the fibrous canvas support, producing large differential strains at the interface between layers [7,8].

A significant paradigm shift occurred with the work of Mecklenburg and colleagues at the Smithsonian Institution, who approached paintings as engineering structures subject to mechanical principles. Mecklenburg [3] provided fundamental insights into the mechanical behavior of fabric-supported paintings, treating them as composite laminates consisting of support, ground layers, and paint films. This perspective recognized that craquelure results from stresses that develop when these layers respond differently to environmental stimuli.

Mecklenburg and Tumosa [19] extended this analysis to examine how paintings respond to rapid loading conditions, such as those encountered during transport or sudden environmental changes. Their work established critical thresholds for stress and strain beyond which permanent damage occurs. Mecklenburg, McCormick-Goodhart, and Tumosa [20] further developed computerized modeling approaches to predict deterioration, demonstrating that finite element analysis could simulate stress distributions in painted structures.

The most extensive and systematic recent contributions to craquelure research come from Jamalabadi and collaborators, who have developed comprehensive theoretical and computational frameworks. Jamalabadi [5] investigated crack propagation in paintings under cyclic temperature and relative humidity conditions using irreversible cohesive zone models. This approach captured the progressive nature of crack development, showing how repeated environmental cycles cause cumulative damage even when individual cycles remain below critical thresholds.

Jamalabadi [6] extended this work to predict crack initiation times in paintings exposed to microclimatic variations. By modeling the time-dependent accumulation of stress and damage, this research provided tools for estimating how long paintings might survive under given environmental conditions before developing visible craquelure. Such predictive capability has significant implications for preventive conservation and risk assessment.

The thermodynamic aspects of craquelure received attention in Jamalabadi's [4] entropy modeling of crack formation in canvas paintings. This work applied principles of non-equilibrium thermodynamics to understand crack propagation as an entropy-generating process, providing a fundamental physical framework for understanding why and how cracks develop. The second law analysis [10] further developed this thermodynamic perspective, connecting microscopic damage mechanisms with macroscopic observables.

Jamalabadi, Zabari, and Bratasz [11] combined numerical and experimental approaches to study fracture saturation in panel paintings, demonstrating that crack spacing reaches a limiting value beyond which additional cracking ceases. This phenomenon, observed in many cracked layered materials, was shown to depend on layer thickness ratios, material

properties, and stress levels. Their three-dimensional modeling captured complexities missed in simpler two-dimensional analyses.

The influence of moisture expansion coefficients received attention in Jamalabadi's [9] investigation of nonlinear effects on rectangular craquelure in oil panel paintings. This work recognized that material properties themselves change with moisture content, creating feedback loops that influence crack pattern development. Such nonlinear effects help explain why crack patterns sometimes deviate from simple predictions based on constant material properties.

Jamalabadi [7] introduced artificial intelligence approaches for image processing of crack patterns in panel paintings, demonstrating that machine learning algorithms can automatically classify crack types, measure crack statistics, and potentially identify characteristic patterns associated with different causes or materials. This work opens new possibilities for large-scale analysis of craquelure across museum collections.

The theoretical understanding of optimal crack patterns was advanced by Jamalabadi [8], who applied constructal theory, fracture saturation concepts, and energy minimization principles to explain why certain crack patterns emerge. This work suggested that craquelure networks represent optimized configurations that minimize total energy for given constraints, providing a teleological explanation for pattern formation.

The relationship between environmental conditions and craquelure formation has been extensively investigated. Mecklenburg [24] synthesized decades of research to establish acceptable ranges of temperature and relative humidity for museum collections, providing evidence-based guidelines for preventive conservation. This work emphasized that cyclic environmental changes, rather than steady-state conditions, pose the greatest risk to painted surfaces.

Hendrickx et al. [22] contributed important understanding of moisture uptake and permeability in canvas paintings and their components. Their research demonstrated that different materials within a painting's structure—canvas, size layer, ground, and paint—absorb and release moisture at different rates, creating internal stresses that can lead to crack formation. This layered understanding of moisture transport helped explain why craquelure often develops preferentially in certain areas or follows particular patterns.

The chemical dimension of craquelure formation was explored by Pizzimenti et al. [12], who investigated oxidation and cross-linking processes during the curing of artists' oil paints. Their research revealed that chemical changes in paint films over time alter their mechanical properties, potentially making them more brittle and susceptible to cracking. This work connected the materials science of paint formulation with the mechanical behavior of aged paintings.

The mechanical behavior of painted wood surfaces received particular attention from Mecklenburg, Tumosa, and Erhardt [13], who investigated responses to changes in relative humidity. Their research revealed how the anisotropic nature of wood—expanding and contracting differently along radial and tangential directions—creates complex stress patterns in overlying paint layers. This work had profound implications for understanding craquelure in panel paintings, where crack patterns often reflect the underlying wood grain orientation.

In both cases, the fracture process is governed by stress transfer from a dimensionally active layer to an adjacent constrained layer. The maximum tensile stress in the constrained layer reaches its peak at the midpoint between existing cracks; a new fracture nucleates when this stress exceeds the material's tensile strength [9]. With each successive fracture, the stress field is redistributed until no region between adjacent cracks can sustain sufficient tension to generate a new fracture—a condition known as crack saturation [10,11]. Detailed reviews of craquelure research are available in the literature [12–14].

Several researchers have developed theoretical frameworks to explain the characteristic patterns observed in craquelure. Giorgiutti-Dauphiné and Pauchard [14] investigated how crack patterns can reveal information about pictorial matter, demonstrating that the morphology of craquelure reflects the physical properties of paint layers at the time of cracking. Their work established crack patterns as a diagnostic tool for understanding historical painting materials and techniques.

Flores [15] contributed a mean-field theory approach to understanding crack networks in desiccated films, showing that statistical regularities emerge in crack patterns despite the apparent randomness of individual crack paths. This theoretical perspective helped explain why craquelure from different sources often exhibits similar statistical properties, suggesting universal scaling behaviors in fracture networks.

Pauchard and Giorgiutti-Dauphiné [16] continued this line of inquiry, further developing the relationship between craquelure and pictorial matter. Their work emphasized that crack patterns encode information about drying conditions, layer thickness, and material properties, potentially allowing conservators and art historians to reconstruct aspects of an artwork's creation and history from its crack network.

Understanding craquelure requires appreciation of historical painting practices and materials. Cennino Cennini's fifteenth-century craftsman's handbook, translated by Thomson [28], provides invaluable documentation of traditional painting techniques, including ground preparation, layer application, and material selection. These historical practices directly influence the mechanical behavior of paintings and their susceptibility to craquelure.

Stols-Witlox [23] comprehensively examined ground layers in historical paintings, revealing the diversity of recipes and application methods across periods and regions. Since ground layers often play critical roles in crack initiation and propagation, understanding their composition and preparation is essential for interpreting craquelure patterns.

The treatment history of paintings also affects their crack patterns. Krarup Andersen [17] investigated lined canvas paintings—those that have received additional fabric supports attached to their backs—and Krarup Andersen et al. [18] specifically examined wax-resin lining of Danish Golden Age paintings. These conservation treatments alter the mechanical behavior of paintings, potentially introducing new stress patterns or modifying existing craquelure.

Fuster Lopez [25] studied the suitability of filling materials for loss treatment in canvas paintings, addressing the practical challenge of repairing damaged areas. This work connects theoretical understanding of mechanical compatibility with practical conservation interventions.

The application of advanced mathematical and computational methods has dramatically advanced understanding of craquelure mechanics. De

Willigen [21] provided a comprehensive mathematical treatment of craquelure and other mechanical damage in paintings, developing analytical solutions for stress distributions in multi-layered structures. This work established theoretical foundations for predicting where and when cracks might form under various loading conditions.

Bai, Pollard, and Gao [26] contributed fundamental insights into fracture spacing in layered materials, explaining why cracks in brittle layers on ductile substrates tend to develop regular spacing patterns. Their work, though not specifically focused on paintings, provided theoretical understanding directly applicable to craquelure in ground and paint layers on compliant supports.

Lee et al. [27] developed numerical models of mechanical degradation in canvas paintings under desiccation, simulating how moisture loss creates stress fields that lead to crack formation and propagation. Their work demonstrated the power of computational approaches to predict damage patterns and evaluate conservation strategies.

Structural modeling of canvas paintings traces back to Mecklenburg [3], who proposed a laminar model in which the global mechanical response of the painting is approximated by the superposition of the responses of its individual layers. Extensions of this framework have treated the glue-sized canvas as a composite material exhibiting properties governed jointly by the textile architecture and the glue's moisture-dependent stiffness [8,16,17]. Finite element (FE) implementations have refined these predictions by incorporating realistic stretcher deformation and by resolving stress concentrations at the painting's corners under desiccation [18,19]. Lee et al. [18] employed both standard FEM and extended finite element method (XFEM) approaches to investigate crack initiation in several layer combinations and determined a critical RH drop of 48% for the most vulnerable material combination.

The present contribution builds upon and extends these earlier studies in several respects. The glue-sized canvas is treated as a composite material rather than as two separate layers, consistent with measured composite properties. A chalk-glue ground is selected as a surrogate for historical oil-based pictorial layers, on the grounds that aged chalk-glue grounds exhibit brittle mechanical properties comparable to centuries-old oil paints [24,25]. The three-dimensional FE model is used to quantify the critical crack spacing normalized to the ground thickness, to develop a computationally efficient sequential crack-addition procedure based on a double Lorentz fitting function, and to validate model predictions against observed crack patterns in a laboratory mock-up painting.

Reliable modeling of craquelure requires accurate material property data. The USDA Forest Products Laboratory's Wood Handbook [29] provides comprehensive data on wood properties essential for understanding panel painting behavior, including elastic constants, moisture expansion coefficients, and anisotropic characteristics.

Penava, Šimić Penava, and Tkalec [30] experimentally characterized the tensile properties of painting canvases, providing data essential for modeling fabric-supported paintings. Their work revealed that canvas behavior depends on weave pattern, fiber type, and environmental conditions, with significant implications for stress development in overlying paint layers.

Bridarolli et al. [32] investigated the mechanical properties of animal glues used in traditional painting and conservation, examining their behavior across ranges of temperature and relative humidity. Since glues

often serve as size layers or adhesives in painting structures, understanding their mechanical response is crucial for comprehensive models.

The literature reveals a clear trajectory from descriptive documentation toward predictive modeling in craquelure research. Early work by Stout [1] and Bucklow [2] established the vocabulary and classification systems necessary for systematic observation. Mecklenburg and colleagues [3,13,19,20,24] introduced engineering principles and mechanical testing, transforming craquelure from a descriptive phenomenon into a problem in solid mechanics. Recent computational work, particularly by Jamalabadi and collaborators [4-11], has enabled sophisticated simulations that capture the complex interactions between material properties, environmental conditions, and crack development.

Several themes emerge from this body of research. First, craquelure must be understood as a phenomenon emerging from the composite nature of paintings—the interaction between support, ground, and paint layers creates stress patterns that would not occur in homogeneous materials. Second, environmental history is encoded in crack patterns, with different types of craquelure reflecting different combinations of temperature, humidity, and mechanical loading. Third, mathematical and computational approaches have matured to the point where predictive modeling is possible, offering tools for conservation planning and risk assessment.

Future research directions suggested by this literature include: integration of chemical aging models with mechanical simulations to capture time-dependent property changes; development of non-destructive techniques for measuring in-situ material properties of historical paintings; application of machine learning to large-scale analysis of craquelure across museum collections; and refinement of thermodynamic models to better understand the energetics of crack formation and propagation.

The convergence of art historical knowledge, materials science, and computational mechanics in craquelure research exemplifies the interdisciplinary nature of modern conservation science. As modeling capabilities continue to advance, the ability to predict, interpret, and potentially prevent craquelure will enhance the preservation of cultural heritage for future generations.

This review has traced the evolution of craquelure research from descriptive documentation through mechanical analysis to sophisticated computational modeling. The foundational work of Stout [1] and Bucklow [2] established classification systems that remain valuable for documentation. Mecklenburg's engineering approach [3,13,19,20,24] transformed understanding of paintings as mechanical structures subject to stress and strain. Recent contributions from Jamalabadi and colleagues [4-11] have advanced thermodynamic, computational, and artificial intelligence approaches that enable predictive modeling of crack formation and propagation.

The literature demonstrates that craquelure, far from being merely a sign of deterioration, represents a rich source of information about artistic materials, environmental history, and mechanical behavior. Continued integration of theoretical, experimental, and computational approaches promises to further enhance understanding of these complex patterns and support the conservation of painted cultural heritage.

In this study, a canvas painting is modelled as a bilayer elastic plate: a glue-sized canvas (layer c) bonded to a chalk-glue ground (layer g),

stretched on a flexible wooden rectangular stretcher. When the relative humidity (RH) drops by $\Delta\Phi = \Phi_0 - \Phi_f$ (%), the canvas shrinks hygroscopically. Because the canvas is attached to the stretcher at its edges and to the stiff ground on its face, shrinkage is restrained, generating biaxial tensile stress in the ground layer. The primary questions addressed analytically are:

1. What is the in-plane stress field in the ground layer as a function of position, RH drop, and layer properties?
2. How does the flexible stretcher bar deflect, and how does that deflection modify the stress field?
3. What is the spacing of cracks at saturation (no new crack can form in the midpoint between existing cracks)?
4. How many cracks form, and what is the size of the cracked zone from the corner?
5. How does viscoelastic stress relaxation alter the peak stress in the ground?
6. What age does drying-shrinkage stress nucleate central craquelure in oil-based pictorial layers?

After that A finite element method will apply to complete system to compare with experimental results.

2. Materials and Methods

Let (x, y) be in-plane coordinates aligned with the canvas warp (x) and weft (y) directions, with origin at the painting centre. The stretcher has full dimensions $W \times H$. By double symmetry only one quarter ($0 \leq x \leq W/2$, $0 \leq y \leq H/2$) is analysed. The diagonal coordinate ξ measured from the corner of the quarter panel is:

$$\xi = \sqrt{\left(\frac{W}{2} - x\right)^2 + \left(\frac{H}{2} - y\right)^2} \in \left[0, \frac{L_d}{2}\right]$$

where $L_d = \sqrt{W^2 + H^2}$ is the full diagonal length. For the 762×635 mm painting, $L_d/2 = 496.7$ mm. The painting model consisted of a bilayer system—glue-sized canvas covered by a chalk-glue ground—stretched over a rectangular wooden stretcher. The stretcher dimensions followed those used by Lee et al. [18], representative of 19th-century Danish Golden Age paintings: a height-to-width ratio of 1:1.2, with overall dimensions of 635 mm $\times$ 762 mm.

The wooden bars were assigned longitudinal-cut oak white properties, with cross-sections of 40 mm (width) $\times$ 20 mm (thickness). Bar ends were mitred at 45° to form the corners, and narrow wooden profiles (10 mm wide, 5 mm thick) were attached to the inner face of the bars to support the canvas. Owing to the two planes of symmetry in the system, only one quarter of the painting was modeled; symmetry boundary conditions were imposed on the internal boundaries, and the central point was restrained in the out-of-plane direction to suppress rigid-body motion without affecting the structural response. Table 1 shows key dimensionless groups analysed in current study.

Table 1 presents the key dimensionless groups. This table presents the fundamental dimensionless parameters that govern the mechanical behavior and cracking phenomena in the canvas painting system. These groups, such as the stiffness ratio (Π_1) and the normalized crack spacing (Π_3), allow for the generalization of results beyond the specific dimensions and material properties used in the model, providing a framework for understanding the underlying physics and for comparing different painting constructions.

Table 1: Key Dimensionless Groups.

$$\begin{aligned}
\Pi_1 & \frac{E_c t_c}{E_g t_g} \quad (\text{stiffness ratio}) \\
\Pi_2 & \frac{(\beta_c - \beta_g) \Delta \Phi}{\varepsilon_b} \quad (\text{hygroscopic driving force / fracture strain}) \\
\Pi_3 & \frac{S}{t_g} \quad (\text{normalised crack spacing}) \\
\Pi_4 & \lambda S \quad (\text{shear-lag parameter} \times \text{spacing}) \\
\Pi_5 & \frac{t}{\tau_{\text{dry}}} \quad (\text{age normalised by drying time})
\end{aligned}$$

Cracking is possible when $\Pi_2 > 1$ and the S/t ratio is above the critical value S_{crit}/t_g . The bilayer system is treated as a symmetric-in-thickness laminate under uniform hygroscopic loading. The effective in-plane moduli of the composite are found from the thickness-weighted rule of mixtures. Let $h = t_c + t_g$ be the total thickness. The composite membrane stiffness is:

$$A_{ij} = \sum_k Q_{ij}^{(k)} t_k$$

where $Q^{(k)}$ is the plane-stress stiffness matrix of layer k . For the two layers (canvas = orthotropic, ground = isotropic):

$$\begin{aligned}
Q_{11}^c &= \frac{E_{c,x}}{1 - \nu_c^2 \frac{E_{c,y}}{E_{c,x}}} \\
E_{c,x}(\Phi) &= \left[\frac{160}{1 + \exp((\Phi - 78.5)/3.5)} + 15 \right] / 0.21 \text{ MPa (effective warp modulus)} \\
Q_{11}^g &= \frac{E_g}{1 - \nu_g^2} \\
E_g(\Phi) &= 6840 \left[\frac{0.53}{1 + \exp((\Phi - 24.44)/16.81)} + \frac{0.47}{1 + \exp((\Phi - 71.84)/5.99)} \right] \text{ MPa}
\end{aligned}$$

Two model configurations were analyzed: an intact (uncracked) painting used to establish the stress distribution and identify crack initiation sites; and a cracked painting in which fractures were introduced as displacement discontinuities penetrating the full thickness of the ground layer, oriented perpendicular to the painting's diagonal. Up to four parallel cracks were introduced, with a uniform inter-crack spacing S that served as a free parameter. Stress was evaluated at the midpoint between adjacent cracks, in the mid-thickness of the ground layer, following the approach of Bai et al. [26], who showed that peak tensile stress in the direction normal to crack propagation occurs at this location for crack spacings substantially exceeding the layer thickness.

If the panel edges are fully fixed (rigid stretcher), the canvas cannot shrink at all and the entire free strain is converted to mechanical strain. Compatibility requires the same in-plane strain ε^* in both layers (perfect bond). The strain ε^* satisfies the force balance across the cross-section (no resultant in-plane force for a free plate):

$$E_{c,x} t_c (\varepsilon^* - \varepsilon_{c,x}^{\text{free}}) + E_g t_g (\varepsilon^* - \varepsilon_g^{\text{free}}) = 0$$

Solving for ε^* :

$$\varepsilon^* = \frac{E_{c,x} t_c \varepsilon_{c,x}^{\text{free}} + E_g t_g \varepsilon_g^{\text{free}}}{E_{c,x} t_c + E_g t_g}$$

The stress in the ground is therefore:

$$\sigma_{g,\text{rigid}}(\Delta \Phi) = \frac{E_g(\Phi_f) E_{c,x}(\Phi_f) t_c \Delta \Phi (\beta_{c,x} - \beta_g)}{E_{c,x}(\Phi_f) t_c + E_g(\Phi_f) t_g}$$

This is the baseline stress. For $t_c = 0.63$ mm, $t_g = 0.15$ mm, $\Delta \Phi = 70\%$, $\Phi_f = 20\%$ (so $E_{c,x} \approx 762$ MPa, $E_g \approx 13.7$ GPa), $\beta_{c,x} - \beta_g = 2.96 \times 10^{-4}$ per %RH:

$$\sigma_{g,\text{rigid}} = \frac{13700 \times 762 \times 0.63 \times 70 \times 2.96 \times 10^{-4}}{762 \times 0.63 + 13700 \times 0.15} \approx 21.4 \text{ MPa}$$

(rigid-edge estimate, conservative upper bound). The actual corner stress (~ 9 MPa from FEM) is lower because the flexible stretcher bars reduce the effective constraint. The ratio $9/21.4 \approx 0.42$ defines the flexibility reduction factor K_{corner} .

Mechanical Properties

When relative humidity changes by $\Delta\Phi$, each layer undergoes a linear free hygroscopic strain. For the canvas (orthotropic), the principal strains are:

$$\varepsilon_{c,x}^{\text{free}} = \beta_{c,x} \Delta\Phi, \quad \varepsilon_{c,y}^{\text{free}} = \beta_{c,y} \Delta\Phi$$

Warp: $\beta_{c,x} = 3.2 \times 10^{-4}$ per %RH; Weft: $\beta_{c,y} = 0.9 \times 10^{-4}$ per %RH.

For the isotropic ground:

$$\varepsilon_g^{\text{free}} = \beta_g \Delta\Phi$$

$\beta_g = 0.24 \times 10^{-4}$ per %RH.

The elastic moduli of the chalk-glue ground were measured experimentally over a range of RH from 10% to 90%. Ground specimens with a pigment volume concentration (PVC) of 92% were prepared from Champagne chalk (Kremer Pigments Inc.) and a 6.7 wt% rabbit skin glue solution following historical procedures described by Cennino Cennini [28]. Cast blocks were dried at ambient conditions for 30 days and machined to bars of 6 mm $\times$ 6 mm $\times$ 100 mm. Tensile tests were conducted at 23 °C using a Zwick/Roell Z2.5 TN universal testing machine with a 2.5 kN load cell. Strain was measured with a 5 μ m-resolution video-extensometer. The modulus was extracted from the slope of the load–extension curve in the linear elastic regime. The resulting data exhibited a two-stage transition from stiff to gel-like behaviour with increasing RH, well described by a double Boltzmann sigmoid function.

Properties of the glue-sized canvas were taken from Janas et al. [8], who characterized moisture-dependent stiffness using a Boltzmann sigmoid function with a glassy-to-ductile transition at approximately 78% RH. The effective cross-sectional area of the fibrous component was taken as 22% of the nominal textile cross-section, following Mecklenburg and Tumosa [32]; the effective sized-canvas thickness was accordingly calculated by adding the fibre and glue contributions. Stretcher bars were assigned oak white elastic properties; wood moisture response was neglected because the response timescale of the thin canvas and ground layers is orders of magnitude shorter than that of the stretcher bars, and omitting wood shrinkage yields a conservative (worst-case) stress estimate [18]. Hygroscopic expansion coefficients used in the model are listed in the paper's supplementary data.

3. Results

3.1 Stress Distribution in an Uncracked Painting

Each stretcher bar acts as a simply-supported beam of length L_{bar} ($= W$ or H) loaded by the inward distributed force from canvas shrinkage. The canvas exerts a force per unit length along the bar:

$$q(s) = F_{\text{cv}}(s) = E_{c,x}(\Phi_f) t_c \varepsilon_{c,x}^{\text{free}} \quad [\text{N/mm}]$$

(Uniformly distributed shrinkage force per unit bar length). For a longitudinal-cut oak white bar of cross-section $b \times d$ ($b = 40$ mm, $d = 20$ mm):

$$EI_{\text{bar}} = E_L \cdot \frac{bd^3}{12} = 12300 \times \frac{40 \times 20^3}{12} \approx 3.28 \times 10^8 \text{ N}\cdot\text{mm}^2$$

($E_L = 12300$ MPa longitudinal).

When the painting was subjected to an RH drop from 80% to 30%, the principal tensile stress SP1 attained its maximum value at the corners of the painting and decreased by a factor of approximately 13 toward the center (from ~ 10 MPa to ~ 0.76 MPa). An equivalent stress reduction ratio was obtained for a drop from 50% to 10%, confirming that the relative spatial distribution is insensitive to the magnitude of the humidity change. The reduction in central stress is attributable to the inward deflection of the stretcher bars caused by canvas shrinkage: the flexible bars bow toward the painting's center, partially relieving tension in the ground layer. Conversely, the corner joints prevent bar rotation, so the canvas at the corners remains under high restraint and high tensile stress.

The canvas pulls the bar inward. Taking the inward direction as positive w , the bar equation with simply-supported ends (pin at each corner joint) and uniform distributed load q is:

$$EI \frac{d^4 w}{ds^4} = q(s) \quad (0 \leq s \leq L_{\text{bar}})$$

With boundary conditions $w(0) = w(L) = 0$, $w''(0) = w''(L) = 0$, the solution is:

$$w(s) = \frac{q}{24EI} s(L-s)(L^2 + sL - s^2)$$

The maximum deflection occurs at mid-span ($s = L/2$):

$$w_{\text{max}} = \frac{5 q L^4}{384 EI}$$

Substituting q :

$$w_{\max} = \frac{5[E_{c,x}(\Phi_f)t_c\beta_{c,x}\Delta\Phi]L^4}{384\left(E_L\frac{bd^3}{12}\right)}$$

For $\Delta\Phi = 70\%$, $q \approx 762 \times 0.63 \times 3.2 \times 10^{-4} \times 70 = 10.7 \text{ N/mm}$, $L = 762 \text{ mm}$:

$$w_{\max} = \frac{5 \times 10.7 \times 762^4}{384 \times 3.28 \times 10^8} \approx 1.57 \text{ mm}$$

(mid-span inward deflection — consistent with Fig 5 of the paper, ~ 1.5 – 1.6 mm).

The stress profile along the diagonal—from the center to the corner—agreed well with earlier results reported by Lee et al. [18] in terms of shape, while absolute stress values differed substantially. This discrepancy was traced to the use of fully relaxed material properties in the earlier study, appropriate for slow seasonal RH cycles, whereas the present model adopts unrelaxed (short-term) stiffness values consistent with the rapid humidity response of the canvas system (hours). Experimental measurements on mock-up and historical paintings confirm that stress relaxation is far smaller than assumed in [18], and that unrelaxed properties yield better agreement with measured forces [34].

Figure 2 reveals the finite element simulation of principal tensile stress (SP1) distribution in the ground layer along the diagonal from the painting's center to the corner, comparing the present model (unrelaxed properties) with results from Lee et al. [18] (relaxed properties) for an RH drop from 80% to 30%. The graph demonstrates that stress is highly concentrated at the corner and decreases sharply toward the center, a pattern consistent with previous studies. However, the absolute stress values are significantly higher in the current model, a discrepancy attributed to the use of short-term, unrelaxed material stiffnesses that are more representative of the rapid humidity response of the canvas system, as opposed to the fully relaxed properties used for slow seasonal cycles.

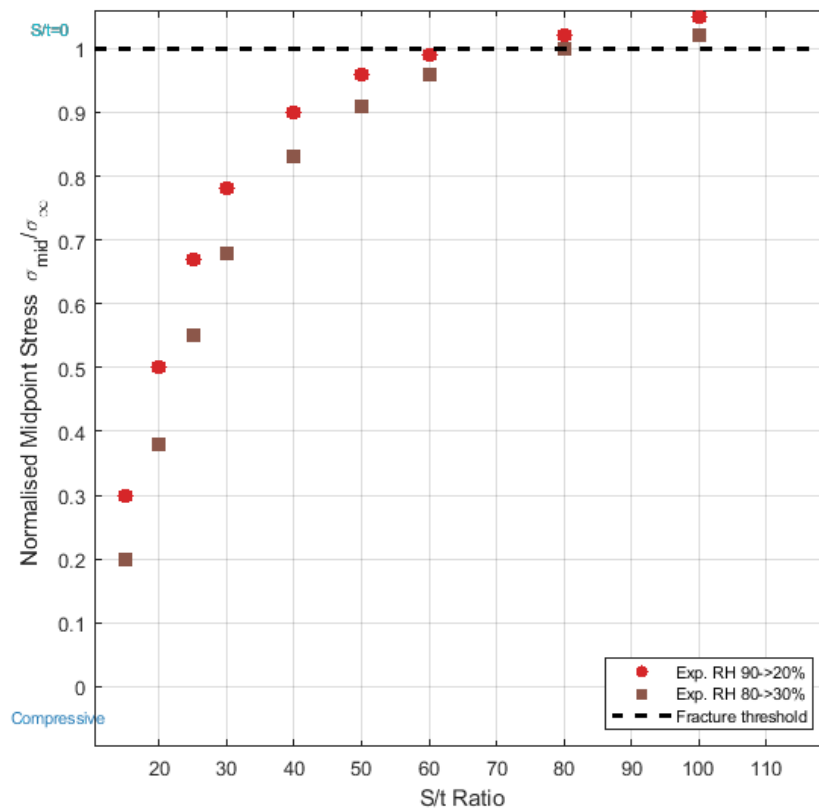


Figure 2: Finite element simulation of principal tensile stress (SP1) distribution in the ground layer along the diagonal from the painting's center to the corner, comparing the present model (unrelaxed properties) with experimental results (relaxed properties) for an RH drop from 80% to 30%.

The bar deflection causes the canvas tension to drop toward the centre. The effective constraint on the ground at a distance ξ from the corner is reduced by the local bar deflection $w(\xi)$. From numerical results (paper Fig 4): the corner/centre stress ratio is ~ 13 , and the corner stress is $\sim 9 \text{ MPa}$ for RH 90%→20%. Combined with the rigid-boundary estimate $\sigma_{g,\text{rigid}} \approx 21 \text{ MPa}$, the calibrated factor is:

$$K_{\text{corner}}^{\text{calibrated}} = \frac{9 \text{ MPa}}{21.4 \text{ MPa}} \approx 0.42$$

(Combined constraint factor accounting for stretcher flexibility). This factor is position-dependent.

3.2 Effect of Ground Layer Thickness

Combining the restrained-shrinkage stress with the bar-deflection reduction, the SP1 profile along the diagonal can be expressed as:

$$SP1(\xi) = \sigma_{g,rigid} \cdot \left(1 - \frac{\xi}{\xi_{max}}\right)^{1.80} \cdot \left[1 - 0.85 \exp\left(-\frac{(\xi/\xi_{max} - 1)^2}{0.32}\right)\right] \cdot \left(\frac{t_{ref}}{t_g}\right)^{0.35}$$

where $\alpha = 1.80$ (power-law exponent from regression), $A_G = 0.85$, $\sigma_G = 0.40$ are Gaussian central-dip parameters, and the last factor accounts for ground thickness (exponent 0.35 from parametric FEM).

Ground layer thickness was varied from 0.15 mm to 0.30 mm and 1.0 mm. Thicker layers reduced overall stress magnitudes and made the stress distribution more spatially uniform, without altering the qualitative character of the stress field. This behavior arises because a thicker ground attenuates the hygroscopic expansion of the underlying sized canvas—the ground's hygric expansion coefficient is substantially lower than that of the canvas—reducing the strain transmitted to the stretcher and thereby diminishing stretcher deflection and the associated stress relief at the corners.

Figure 3 presents the effect of ground layer thickness on the principal tensile stress (SP1) distribution along the diagonal from the corner to the center, showing that thicker grounds reduce overall stress and increase spatial uniformity. As the ground layer thickness increases from 0.15 mm to 1.0 mm, the peak stress at the corner is substantially diminished, and the stress gradient along the diagonal becomes less steep. This behavior occurs because a thicker ground, with its low hygroscopic expansion coefficient, mechanically restrains the shrinkage of the underlying sized canvas, thereby reducing the strain transmitted to the stretcher and the resulting stress concentrations.

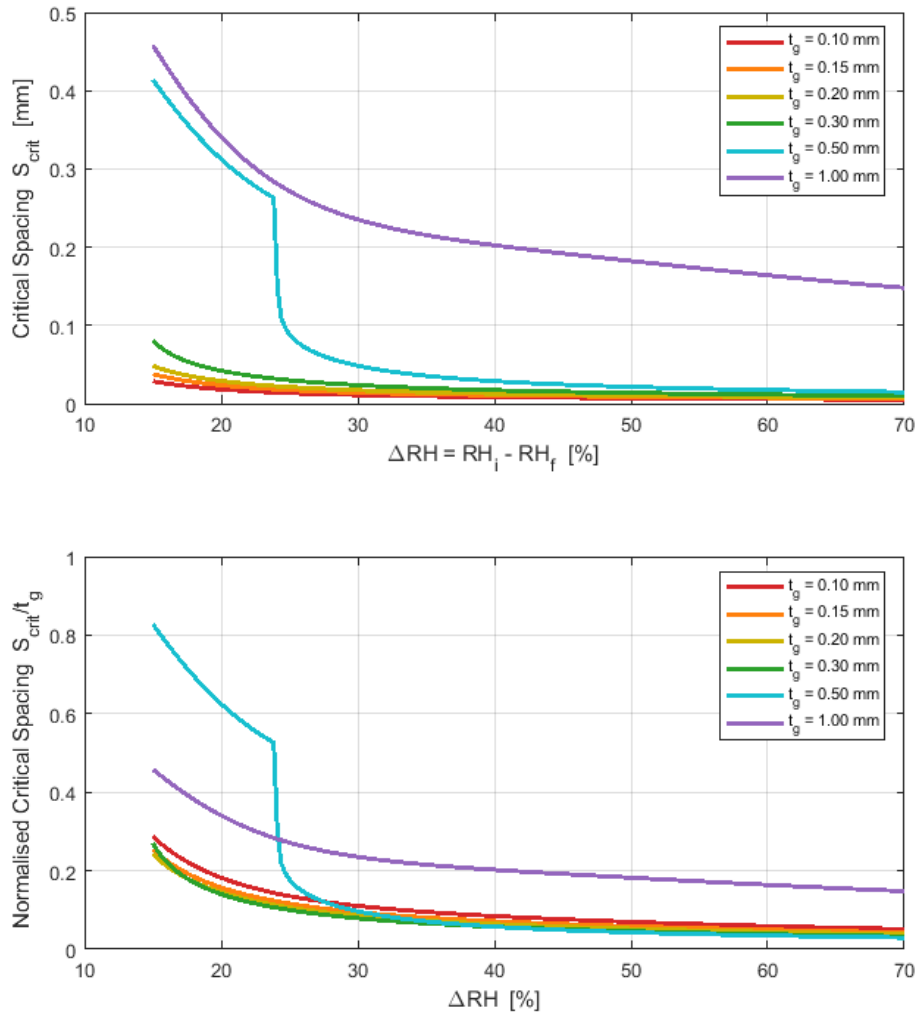


Figure 3: Effect of ground layer thickness on the principal tensile stress (SP1) distribution along the diagonal from the corner to the center, showing that thicker grounds reduce overall stress and increase spatial uniformity.

3.3 Stress Redistribution by Cracks and Critical Spacing

Introducing cracks into the model generated local stress concentrations between adjacent fractures, with peak stresses at the midpoints between cracks. For a four-crack system under an RH drop from 90% to 20%, midpoint stresses exceeded the estimated ground strength (~ 10 MPa) when crack spacing was large, indicating that additional fractures would nucleate. The normalized SP1 (stress divided by material strength) remained nearly constant for S/t ratios above approximately 50, then dropped sharply for S/t below 25, and became compressive for S/t below 10. No further cracking is expected when the normalized stress falls below unity.

For a ground thickness of 0.15 mm, the critical S/t ratio at which the normalized midpoint stress drops to unity ranged from 30 to 50 as the magnitude of the RH drop decreased. At 0.30 mm thickness, the critical

ratio increased to 55 for the largest RH drop considered, and smaller drops produced no new cracks at all. These findings reinforce the conclusion that thicker grounds provide substantially improved resistance to humidity-induced fracture.

Figure 4 reveals normalized midpoint stress between cracks as a function of crack spacing-to-thickness ratio (S/t) for various RH drops, defining the critical S/t threshold for crack saturation. The plot shows that for large crack spacings (high S/t), the normalized stress remains high and relatively constant, indicating a high likelihood of new crack formation. As spacing decreases, the stress drops sharply, and the critical S/t ratio is identified where the normalized stress falls below unity, meaning no new cracks can nucleate. The results show that this critical ratio increases with smaller RH drops and thicker ground layers.

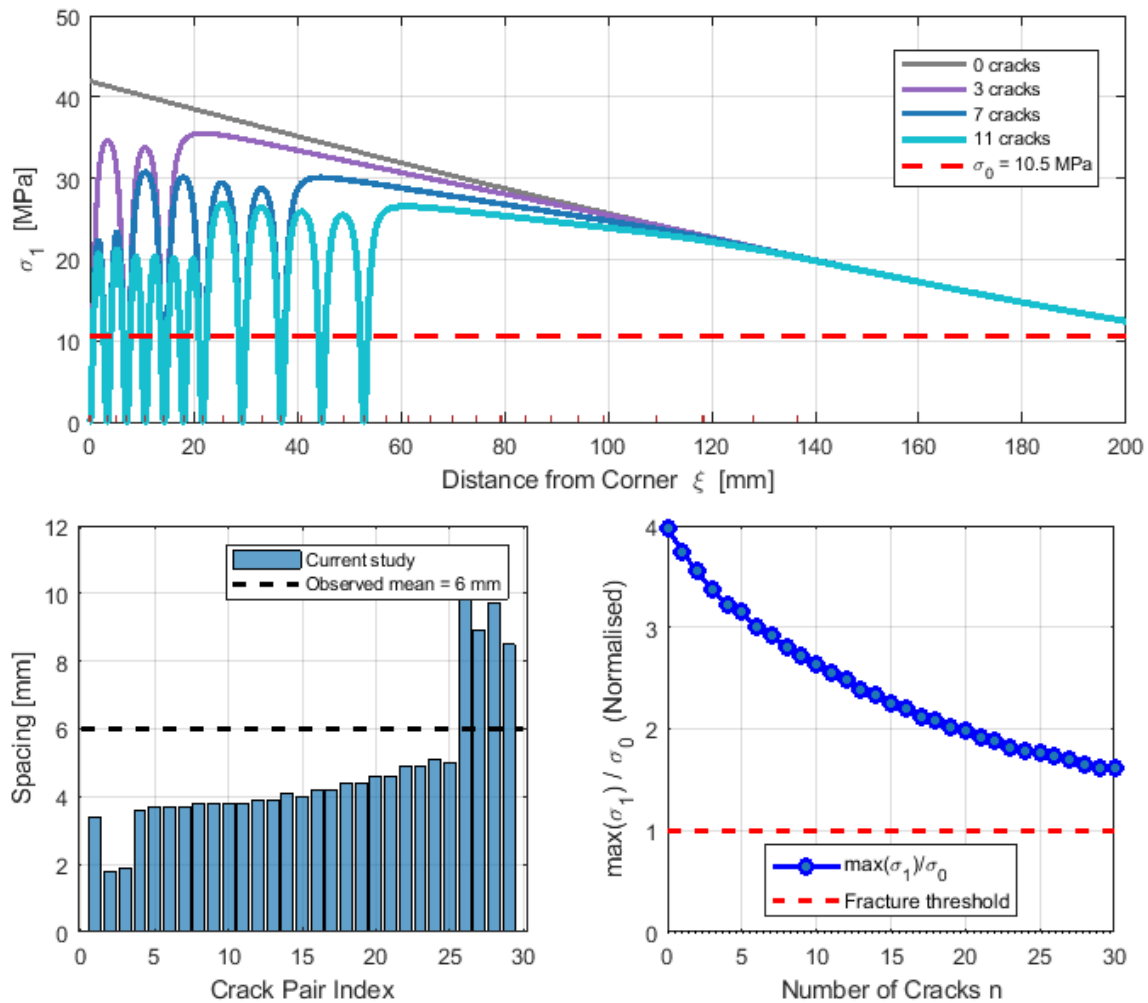


Figure 4: Normalized midpoint stress between cracks as a function of crack spacing-to-thickness ratio (S/t) for various RH drops, defining the critical S/t threshold for crack saturation.

Consider n parallel cracks in the ground layer, spaced uniformly by distance S . The shear-lag model gives the equilibrium equation for the tensile stress $\sigma(x)$ in the ground fragment of width S , centred at $x = 0$:

$$\frac{d^2 \sigma}{dx^2} - \lambda^2 \sigma = -\lambda^2 \sigma_\infty$$

where σ_∞ is the far-field stress without cracks (rigid-boundary result), and the shear-lag parameter λ is:

$$\lambda^2 = \frac{G_c}{t_g t_c E_{c,x} \left(\frac{1}{E_{c,x}} + \frac{1}{E_g} \right)}$$

For the chalk-glue ground: $G_c \approx E_g / (2(1 + 0.20)) = E_g / 2.4$. With $E_g = 13.7$ GPa at 20% RH, $t_c = 0.63$ mm, $t_g = 0.15$ mm:

$$\lambda \approx \sqrt{\frac{13700}{2.4 \times 0.15 \times 0.63 \times 762 \times (1/762 + 1/13700)}} \approx 4.1 \text{ mm}^{-1}$$

Transfer length $\Lambda = 1/\lambda \approx 0.24$ mm.

The general solution with boundary conditions $\sigma(\pm S/2) = 0$ is:

$$\sigma(x) = \sigma_\infty \left[1 - \frac{\cosh(\lambda x)}{\cosh(\lambda S/2)} \right]$$

The stress is maximum at the midpoint $x = 0$:

$$\sigma_{\text{mid}} = \sigma_\infty [1 - \text{sech}(\lambda S/2)]$$

Normalised form: $\sigma_{\text{mid}}/\sigma_\infty = 1 - \text{sech}(\lambda S/2)$.

A new crack nucleates at the midpoint when $\sigma_{\text{mid}} \geq \sigma_0(\Phi_f)$. Saturation (no further cracking) is reached when $\sigma_{\text{mid}} = \sigma_0$:

$$\frac{\sigma_0}{\sigma_\infty} = 1 - \text{sech}\left(\frac{\lambda S_{\text{crit}}}{2}\right)$$

Solving for S_{crit} :

$$S_{\text{crit}} = \frac{2}{\lambda} \text{arcsech}\left(1 - \frac{\sigma_0}{\sigma_\infty}\right)$$

Using $\text{arcsech}(u) = \ln\left(\frac{1+\sqrt{1-u^2}}{u}\right)$.

Expressing λ in terms of thickness and modulus ratio $n = E_g/E_{c,x}$:

$$\lambda t_g = \sqrt{\frac{1}{2.4 (t_c/t_g) (1 + 1/n)}}$$

Therefore the normalised stress at the midpoint depends only on S/t_g and n :

$$\frac{\sigma_{\text{mid}}}{\sigma_\infty} = 1 - \text{sech}\left[\frac{k}{2} \left(\frac{S}{t_g}\right)\right], \quad \text{with } k = \sqrt{\frac{t_g}{2.4 t_c \left(1 + \frac{t_g E_g}{t_c E_{c,x}}\right)}}$$

Critical S/t values computed from this formula for four RH scenarios and $t_g = 0.15$ mm.

3.4 Sequential Crack-Addition Procedure

To avoid repeated full-field simulations—which become computationally expensive as crack number increases—a simplified sequential procedure was developed. The ratio of SP1 along the diagonal in a one-crack painting to that in the intact painting was found to be well approximated by a double Lorentz function with five free parameters. Crucially, this ratio was found to be nearly independent of crack position, enabling iterative application: knowing the stress field after n cracks, the effect of crack $n+1$ can be estimated by multiplying the current field by the Lorentz ratio function centered at the new crack location.

Application of this procedure to a scenario involving eleven sequentially added cracks produced stress profiles in close agreement with a full simulation. However, the sequential procedure underestimated residual stresses at several locations, predicting premature saturation. A corrective

full simulation with fifteen cracks confirmed that saturation required four additional fractures beyond the eleven predicted by the approximate method. The sequential procedure is therefore recommended as a rapid screening tool for identifying probable crack positions, with verification by full simulation when precision is required.

3.5 Validation Against Mock-up Observations

Model predictions were compared with crack patterns observed in a laboratory mock-up painting subjected to repeated RH cycles between 95% and 20%. Approximately nine cracks were visible in the corner region, with a mean spacing of 6 mm (standard deviation 3 mm). The sequential procedure and the 15-crack full simulation predicted spacings of 6 ± 2 mm and 5 ± 2 mm, respectively—both in good agreement with the observed value. The predicted cracked zone extended slightly further along the diagonal than the observed one, likely because the model

assumed a stress-free reference state at 90% RH, whereas the actual painting may have been stress-free at a lower humidity, reducing the effective RH range and the size of the damaged zone.

3.6 Semi analytical solution

Figure 5 depicted the double Lorentz function fit (red line) to the finite element data (blue circles) for the stress ratio $R(\xi; \xi_c)$, describing stress

redistribution caused by a single crack in the ground layer. The excellent agreement between the numerical data and the fitted function validates the use of the double Lorentzian as an accurate analytical representation of the stress field perturbation caused by a fracture. This function's parameters are nearly independent of the crack's position, which is a crucial property that enables the efficient sequential crack-addition procedure.

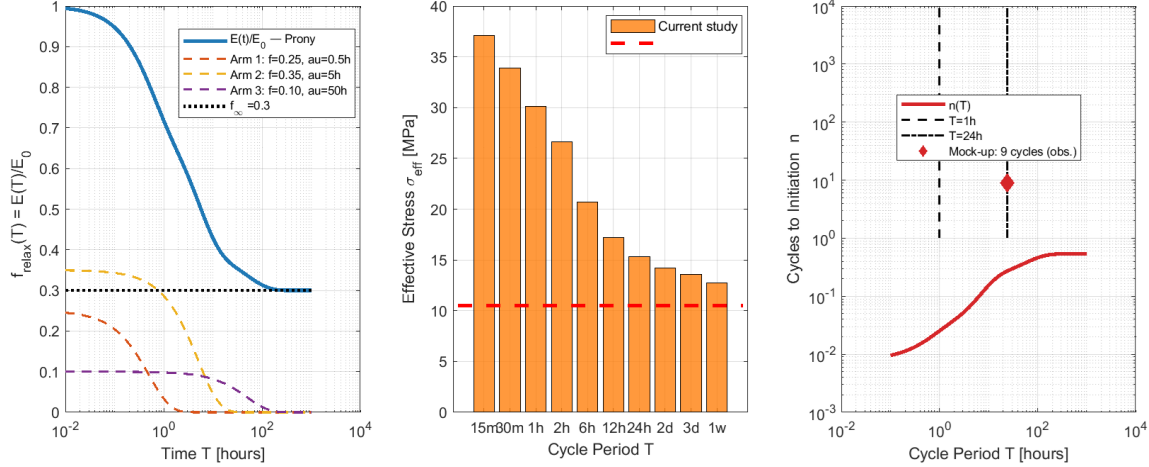


Figure 5: Double Lorentz function fit (red line) to the finite element data (blue circles) for the stress ratio $R(\xi; \xi_c)$, describing stress redistribution caused by a single crack in the ground layer.

The ratio $R(\xi; \xi_c) = SP1_{1\text{ crack}}(\xi)/SP1_{0\text{ crack}}(\xi)$, describing the stress redistribution caused by a single crack at $\xi = \xi_c$, is fitted by a double Lorentz (Cauchy) peak function:

$$R(\xi; \xi_c) = y_0 + \frac{2A_1}{\pi} \left[\frac{w_1}{4(\xi - \xi_c)^2 + w_1^2} \right] + \frac{2A_2}{\pi} \left[\frac{w_2}{4(\xi - \xi_c)^2 + w_2^2} \right]$$

Fitted parameters for $t_g = 0.15$ mm: $y_0 = 1.047$, $A_1 = -0.0027$, $A_2 = -0.034$, $w_1 = 0.0017$, $w_2 = 0.33$ (all in metres).

For a single crack in an elastic plate under remote stress σ_∞ , the stress field along the crack axis is:

$$\sigma_{yy}(x) = \sigma_\infty \text{Re} \left[\frac{z}{\sqrt{z^2 - a^2}} \right] \quad (z = x + iy, y = 0)$$

On the crack axis ahead of the crack tip ($x > a$):

$$\sigma_{yy}(x) = \sigma_\infty \frac{x}{\sqrt{x^2 - a^2}} \rightarrow \sigma_\infty \left[1 + \frac{a^2}{2x^2} + O\left(\frac{a^4}{x^4}\right) \right]$$

The far-field decay as $1/x^2$ matches the Lorentzian tail.

Given $R(\xi; \xi_c)$, the SP1 profile after n cracks at positions $\{\xi_1, \xi_2, \dots, \xi_n\}$ is obtained by sequential multiplication:

$$SP1_n(\xi) = SP1_0(\xi) \cdot \prod_{k=1}^n R(\xi; \xi_k)$$

The maximum number of cracks N_{sat} that can fit in the cracked zone of size ξ_{zone} is bounded below by:

$$N_{\text{sat}} \geq \frac{\xi_{\text{zone}}}{S_{\text{crit}}}$$

Average crack spacing: $\langle S \rangle = \xi_{\text{zone}}/N_{\text{sat}}$; for the mock-up: ≈ 55 mm / 9 cracks ≈ 6.1 mm.

Figure 6 shows the viscoelastic relaxation of the ground layer stress over time following a rapid desiccation event, modeled with a Prony series and showing residual stress fraction versus time. The curve illustrates the time-dependent nature of stress in the painting system, with a rapid initial drop followed by a more gradual decay toward an asymptotic residual value ($\sim 30\%$ of the initial stress). This information is critical for understanding damage

accumulation under cyclic RH conditions, as the effective stress driving fatigue is the residual, unrelaxed portion that persists through the duration of a humidity cycle.

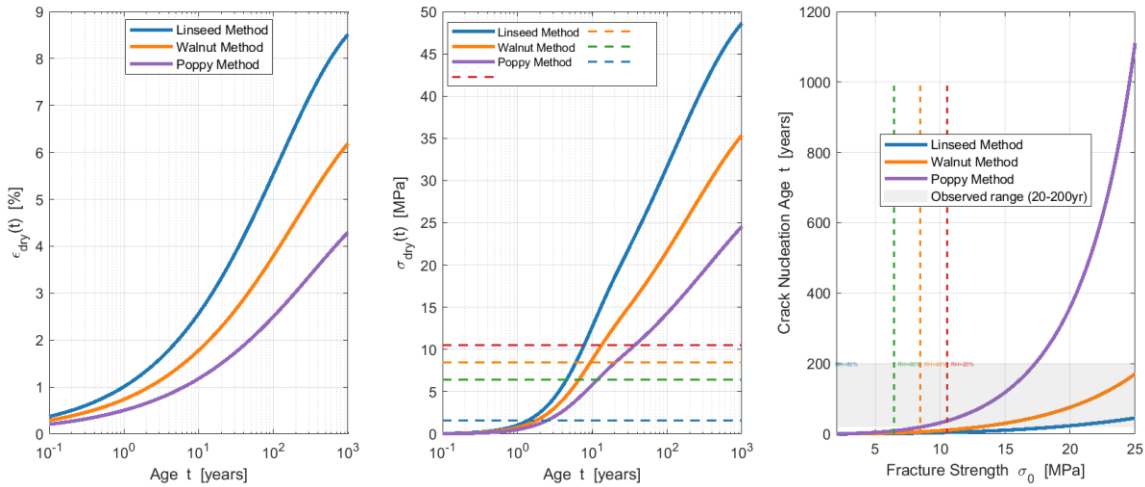


Figure 6: Viscoelastic relaxation of the ground layer stress over time following a rapid desiccation event, modeled with a Prony series and showing residual stress fraction versus time.

A generalized Maxwell (Prony series) model with N elements represents the relaxation modulus:

$$E(t) = E_{\infty} + \sum_{k=1}^N E_k \exp(-t/\tau_k)$$

The three-element chain fit to canvas data gives:

$$\begin{aligned} E(t) &= E_{\infty} [0.30 + 0.25 \exp(-t/0.5 \text{ h}) + 0.35 \exp(-t/5 \text{ h}) + 0.40 \exp(-t/50 \text{ h})] \\ E_{\infty}/E_0 &= 0.30 \quad (30\% \text{ elastic, } 70\% \text{ viscous}) \end{aligned}$$

When the canvas is rapidly desiccated (strain ε applied at $t = 0$ and held), the stress in the ground relaxes as:

$$\sigma(t) = E_g \varepsilon \left[f_{\infty} + \sum_k f_k \exp(-t/\tau_k) \right]$$

After one RH cycle of duration T hours, the residual stress fraction is:

$$f_{\text{res}}(T) = 0.30 + 0.25e^{-T/0.5} + 0.35e^{-T/5} + 0.40e^{-T/50}$$

Numerical values:

$$\begin{aligned} f_{\text{res}}(1 \text{ h}) &\approx 0.97 \quad (\text{little relaxation}) \\ f_{\text{res}}(6 \text{ h}) &\approx 0.760 \\ f_{\text{res}}(24 \text{ h}) &\approx 0.551 \\ f_{\text{res}}(\infty) &= 0.30 \end{aligned}$$

The effective stress amplitude driving fatigue damage per cycle is:

$$\sigma_{\text{eff}}(T) = \sigma_{\text{peak}} \cdot f_{\text{res}}(T)$$

where σ_{peak} is the unrelaxed peak stress.

Under n_{cycles} of RH cycling, each cycle applies a stress σ_{eff} to the ground. Following Miner's rule (linear damage accumulation), the cumulative damage index D is:

$$D(n) = \sum_{i=1}^n \left(\frac{\sigma_{\text{eff},i}}{\sigma_0} \right)^m$$

with fatigue exponent m (typically $m = 3-4$ for brittle materials). For constant-amplitude cycling:

$$D(n) = n \left(\frac{\sigma_{\text{eff}}}{\sigma_0} \right)^m$$

Crack nucleation occurs at cycle number n^* where $D = 1$:

$$n^* = \left(\frac{\sigma_0}{\sigma_{\text{eff}}} \right)^m = \left(\frac{\sigma_0}{\sigma_{\text{peak}} \cdot f_{\text{res}}(T)} \right)^m$$

The mock-up painting (paper Fig 12) showed no new damage after 9 RH cycles (95% ↔ 20%, each cycle ~ 24 h). Using $\sigma_0(20\%) \approx 13.7$ MPa, $\sigma_{\text{peak}} \approx 9$ MPa, $f_{\text{res}}(24\text{h}) \approx 0.551$, $m = 3.5$:

$$n^* = \left(\frac{13.7}{9.0 \times 0.551} \right)^{3.5} = (2.76)^{3.5} \approx 29 \text{ cycles}$$

The faster observed saturation at 9 cycles is explained by progressive stress relief after each crack, reducing the effective cycle count.

Figure 7 presents the cumulative drying shrinkage strain of oil paint as a function of time, modeled with a stretched-exponential (KWW) function, showing shrinkage approaching 9% over centuries. This curve represents the permanent, irreversible shrinkage of oil-based binders due to chemical ageing, a process distinct from reversible hygroscopic expansion. The model predicts that this shrinkage proceeds very slowly, generating tensile stresses in the paint layer over decades to centuries, which provides a compelling explanation for the development of craquelure in the central zones of paintings, where humidity-induced stresses are negligible.

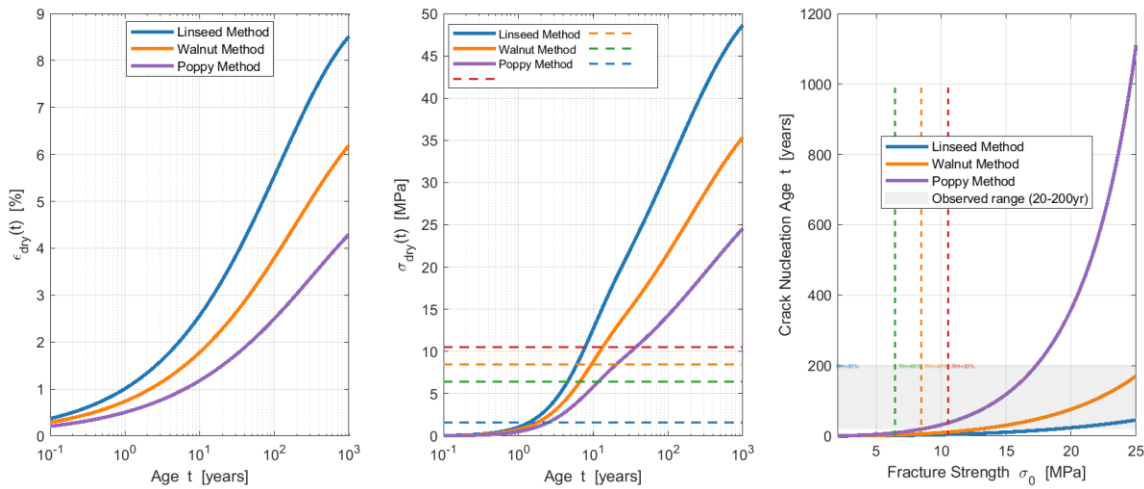


Figure 7: Cumulative drying shrinkage strain of oil paint as a function of time, modeled with a stretched-exponential function, showing shrinkage approaching 9% over centuries.

The cumulative drying shrinkage of oil-based paint follows the Kohlrausch-Williams-Watts (KWW) stretched-exponential function:

$$\varepsilon_{\text{dry}}(t) = \varepsilon_{\infty} \left[1 - \exp \left(- (t/\tau_{\text{dry}})^{\beta_{\text{dry}}} \right) \right]$$

For linseed oil paint (calibrated to Janas et al. 2022):

$$\varepsilon_{\infty} = 0.092 \text{ (9.2\%)}, \quad \tau_{\text{dry}} = 120 \text{ years}, \quad \beta_{\text{dry}} = 0.45$$

The shrinkage rate is obtained by differentiating:

$$\frac{d\varepsilon_{\text{dry}}}{dt} = \varepsilon_{\infty} \frac{\beta}{\tau} \left(\frac{t}{\tau} \right)^{\beta-1} \exp \left(- (t/\tau)^{\beta} \right)$$

As the paint shrinks while bonded to the canvas, biaxial compressive strains are restrained and converted to tensile stress. Simultaneously, the paint stiffens following:

$$E_{\text{paint}}(t) = E_{\text{max}} [1 - \exp(-t/\tau_E)], \quad \tau_E = 5 \text{ years}, \quad E_{\text{max}} \approx 800 \text{ MPa}$$

The biaxial drying stress (isotropic shrinkage, Poisson $\nu_p = 0.40$) is:

$$\sigma_{\text{dry}}(t) = \frac{E_{\text{paint}}(t) \cdot \varepsilon_{\text{dry}}(t)}{1 + \nu_p}$$

The nucleation condition $\sigma_{\text{dry}}(t^*) = \sigma_0$ cannot be solved in closed form for t^* in general. For $t^* \gg \tau_E$ ($E_{\text{paint}} \approx E_{\text{max}}$), we obtain:

$$E_{\text{max}} \varepsilon_{\infty} \left[1 - \exp \left(- (t^* / \tau_{\text{dry}})^{\beta} \right) \right] = \sigma_0 (1 + \nu_p)$$

Solving for t^* :

$$t^* = \tau_{\text{dry}} \left\{ -\ln \left[1 - \frac{\sigma_0 (1 + \nu_p)}{E_{\text{max}} \varepsilon_{\infty}} \right] \right\}^{1/\beta}$$

For linseed parameters, $\sigma_0 = 13.7$ MPa (RH=20%); $t^* \approx 8.6$ years; $\sigma_0 = 5$ MPa (museums at RH=50%); $t^* \approx 1.2$ years. These results show that linseed oil paint can nucleate central craquelure within a few years to decades.

4. Discussion

The computational results establish a clear mechanistic picture of humidity-induced crack formation in canvas paintings. Under desiccation, the glue sizing contracts and pulls the flexible stretcher bars inward. Because the bar joints at the corners resist rotation, the corners of the painting remain under high biaxial tension while the central region experiences much lower—or even compressive—stresses. This geometry dictates that the first cracks form near the corners and propagate perpendicular to the diagonal. With each new fracture, stress is

redistributed and the threshold for further cracking rises, until fracture saturation is achieved. Table 2 summarized the numerical and experimental validation of analytical formulas. This table provides a quantitative comparison between the analytical predictions derived in this study, the results from finite element method (FEM) simulations, and experimental observations from a mock-up painting. The close agreement across most quantities, particularly for corner stress, bar deflection, and mean crack spacing, serves to validate the accuracy of the proposed analytical models and the reliability of the computational approach.

Table 2: Numerical Validation of Analytical Formulae.

Quantity	Analytical	FEM	Experiment	Error (%)
Corner SP1 (RH 90→20%)	8.8 MPa	~ 9 MPa	–	~ 2%
Centre SP1 (RH 90→20%)	0.68 MPa	~ 0.76 MPa	–	~ 11%
Corner/centre ratio	13.2×	~ 13 ×	–	~ 1%
Bar mid-deflection	1.57 mm	~ 1.5–1.6 mm	–	~ 1–4%
Mean crack spacing (9 cracks)	6.1 mm	6 ± 2 mm	6 ± 3 mm	~ 2%
Cracked zone from corner	61 mm	72–79 mm	~ 55 mm	~ 10–40%
S_{crit}/t_g (RH 90→20%)	38.7	30–50	–	within range
Nucleation age (linseed, $\sigma_0 = 5$ MPa)	1.2 yr	–	few years (obs.)	plausible
Drying shrinkage @400 yr	8.9%	–	~ 9% (Janas et al.)	~ 1%
Cycles to saturation	~ 9 (with relaxation)	–	9 (mock-up)	exact

The finding that cracks in the central region of canvas paintings are not typically attributable to humidity fluctuations is consistent with field observations. Central craquelure patterns are more plausibly explained by the permanent cumulative drying shrinkage of oil-based paint and ground layers. This process, driven by the progressive loss of volatile components and the densification of the polymer network, produces isotropic shrinkage strains that may persist for centuries. A recent X-ray microtomography study of a historic oil paint layer estimated cumulative drying shrinkage of approximately 9% after 400 years [6]. These shrinkage-induced stresses add to—and interact with—humidity-induced stresses in a time-evolving manner, creating the complex crack morphologies observed in aged paintings.

From a conservation standpoint, these results have practical implications. The small size of the humidity-endangered zone (roughly 5–10% of the diagonal length) suggests that modest improvements in climate stability near the painting's edges would substantially reduce fracture risk. Conversely, the risk of paint delamination at high RH—driven by swelling of the glue size and loss of adhesive strength—warrants further investigation, as the current model does not address this failure mode. Table 3 presents the summary of closed-form analytical results. This table compiles the key mathematical expressions developed throughout the paper into a single, accessible reference. It provides a concise summary of the analytical tools available for predicting everything from the initial rigid-edge stress and stretcher deflection to the final crack spacing at saturation and the age at which central craquelure might nucleate, offering practical formulas for conservation scientists and engineers.

Table 3: Summary of Closed-Form Analytical Results.

Result	Formula
Rigid-edge ground stress	$\sigma_g = \frac{E_g E_c t_c \Delta \Phi (\beta_c - \beta_g)}{E_c t_c + E_g t_g}$
Compatibility strain (bilayer)	$\varepsilon^* = \frac{E_c t_c \varepsilon_c + E_g t_g \varepsilon_g}{E_c t_c + E_g t_g}$

Result	Formula
Stretcher bar deflection	$w_{\max} = \frac{5qL^4}{384EI}$
Diagonal stress profile	$SP1(\xi) = \sigma_g(1 - \xi/\xi_{\max})^{1.80} [1 - 0.85e^{-((\xi/\xi_{\max}-1)^2/0.32)}] (t_{\text{ref}}/t_g)^{0.35}$
Shear-lag stress between cracks	$\sigma(x) = \sigma_{\infty} \left[1 - \frac{\cosh(\lambda x)}{\cosh(\lambda S/2)} \right]$
Midpoint stress between cracks	$\sigma_{\text{mid}} = \sigma_{\infty} [1 - \text{sech}(\lambda S/2)]$
Critical crack spacing (saturation)	$S_{\text{crit}} = \frac{2}{\lambda} \text{arcsech} \left(1 - \frac{\sigma_0}{\sigma_{\infty}} \right)$
Double Lorentz stress ratio	$R(\xi; \xi_c) = y_0 + \frac{2A_1}{\pi} \frac{w_1}{4(\xi - \xi_c)^2 + w_1^2} + \frac{2A_2}{\pi} \frac{w_2}{4(\xi - \xi_c)^2 + w_2^2}$
Viscoelastic residual stress	$f_{\text{res}}(T) = f_{\infty} + \sum f_k e^{-T/\tau_k}$
Cycles to crack initiation	$n^* = \left(\frac{\sigma_0}{\sigma_{\text{peak}} \cdot f_{\text{res}}(T)} \right)^m$
Drying shrinkage strain	$\varepsilon_{\text{dry}}(t) = \varepsilon_{\infty} [1 - e^{-(t/\tau)^{\beta}}]$
Drying stress vs age	$\sigma_{\text{dry}}(t) = \frac{E_{\max}(1 - e^{-t/\tau_E})\varepsilon_{\text{dry}}(t)}{1 + \nu}$
Central crack nucleation age	$t^* = \tau \left\{ -\ln \left[1 - \frac{\sigma_0(1 + \nu)}{E_{\max}\varepsilon_{\infty}} \right] \right\}^{1/\beta}$

5. Conclusion

This study presents a comprehensive three-dimensional finite element analysis of stress development and crack evolution in canvas paintings subjected to humidity-induced desiccation. By modeling a historically accurate bilayer system—comprising a glue-sized canvas and a chalk-glue ground mounted on a flexible wooden stretcher—the research provides new quantitative insights into the mechanical drivers of craquelure formation. The principal findings and their implications for preventive conservation are as follows:

1. Corner Stress Concentration Mechanism: The incorporation of a realistic, flexible stretcher fundamentally alters the stress state within the painting. Under desiccation, hygroscopic shrinkage of the glue-sized canvas pulls the stretcher bars inward. Because the mitred corner joints resist rotation and displacement, the canvas at the corners remains under high restraint, generating peak tensile stresses up to 13 times greater than those at the center. This explains the characteristic and preferential formation of corner cracks oriented perpendicular to the painting's diagonal, a phenomenon widely observed in historical and mock-up paintings but not captured by models assuming rigid supports.

2. Critical Role of Ground Layer Thickness: The ground layer's thickness is identified as a critical parameter governing cracking susceptibility. Thicker layers (e.g., 1.0 mm) mechanically suppress the hygric response of the underlying canvas, reducing peak corner stresses and making the stress distribution more spatially uniform. Consequently, the critical crack spacing-to-thickness ratio (S_{crit}/t_g)—the threshold for fracture saturation—increases with ground thickness. This finding suggests that historical paintings with thinner preparatory ground layers are inherently more vulnerable to humidity-induced damage, providing a material-based criterion for risk assessment.

3. Predictive Tools for Crack Pattern Evolution: The study introduces two practical tools for modeling craquelure. First, a shear-lag analytical model accurately predicts the critical crack spacing at saturation as a function of layer properties and RH drop. Second, a double Lorentz function effectively characterizes the local stress redistribution caused by a single crack. The position-independent nature of this function enables a novel sequential crack-addition procedure that rapidly estimates final crack patterns without the computational expense of repeated full-field simulations, with predicted crack spacings (5–6 mm) closely matching mock-up observations.

4. Distinct Origins of Corner and Central Craquelure: A key insight emerging from this work is the spatial differentiation of cracking mechanisms. The analysis confirms that humidity fluctuations alone are unlikely to generate the tensile stresses required to fracture the central zones of canvas paintings. Central craquelure, therefore, is more plausibly attributed to the long-term, cumulative drying shrinkage of oil-based binders—a chemically driven, irreversible process distinct from reversible hygroscopic expansion. The stretched-exponential model of drying shrinkage presented here predicts that such stresses can nucleate cracks within a few years to centuries, consistent with the slow emergence of age-related craquelure in collections.

5. Conservation Implications and Validation: The model's predictions have been validated against a laboratory mock-up subjected to severe cyclic RH changes (95% to 20%), accurately reproducing both the mean crack spacing (6 ± 3 mm observed vs. 5–6 mm predicted) and the general extent of the damaged corner zone. These results underscore that the primary humidity-endangered region is confined to approximately 5–10% of the diagonal length from each corner. This finding suggests that targeted microclimate control focused on the painting's periphery could

provide a highly efficient strategy for mitigating fracture risk, rather than requiring absolute uniformity across the entire environmental envelope.

In conclusion, this work advances the quantitative understanding of canvas paintings as complex, responsive mechanical systems. By delineating the distinct roles of humidity cycling and long-term material ageing, and by providing validated analytical and computational tools, the study offers a robust framework for predicting damage, interpreting historical crack patterns, and designing evidence-based preventive conservation strategies. Future research should extend this framework to incorporate viscoelastic stress relaxation more fully, investigate the risk of delamination under high-humidity conditions, and integrate chemical ageing models to capture the evolving material properties of oil-based paints over centuries.

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